A Novel SAR Imaging Algorithm Based on Compressed Sensing

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Abstract

High speed A/D sampling and large scale data storage are two basic challenges of the high resolution SAR system .The developing of radar system is limited by these two challenges under the Nyquist sampling theory. Compressed sensing (CS) is a new approach of sparse signals recovered beyond the constraints of Nyquist sampling technique. With the consideration of these problems that might happen and the advantage of CS theory, a novel SAR image processing algorithm based on compressive sensing was proposed in this paper. Using the data whose sampling rate is lower than the required Nyquist sampling rate, the CS-based algorithm operates at range and azimuth dimensional respectively. Experimental results show the presented algorithm based on compressed sensing have a better performance than the conventional SAR algorithm even with only smaller samples, and also indicate that the presented algorithm is robustness with existence of serious noise

Keywords: compressive sensing; Synthetic Aperture Radar; compressed sensing.

1. Introduction

For Synthetic aperture radar (SAR) can obtain high resolution images of illuminated scene under all weather circumstances，it has become an important imaging and detecting tool for many remote sensing applications in military and civilian fields, including topographic mapping, target identification, and flight navigation etc. However, the development of high resolution radar system is restricted by the Nyquist theory. The resolution of SAR system is limited by the bandwidth of transmitted signal and the length of the antenna. Moreover, according to Nyquist sampling theory, a large amount of samples will be collected in high resolution SAR system which leads to the burden of the system.

In recent years, CS theory attracts more and more attention in signal processing field. This theorem states that a compressible unknown signal can be recovered from incomplete sets of linear measurements by a specifically designed nonlinear recovery algorithm [1, 2]. CS offers many advantages that may be amazing to be used into signal processing, for instance, the possibility of hardware simplification, reduction of data acquisition times, ability of achieving high resolution, and data compression, etc.

Inspired by the idea of CS, R.Baraniuk et al. proposed the radar imaging system based on compressed sensing for the first time [3]. In the past few years, many applications based on compressive sensing have been proposed in radar signal processing area by researchers , such as ISAR imaging[4,5], GPR(ground penetrating radar) imaging[6],MIMO(Multiple Input Multiple Output)radar signal processing[7,8], ultra-wideband radar imaging[9],etc . In the applications of SAR system, CS-based SAR imaging algorithm was proposed in reference[10] , there are also some significant works done on the estimation of moving target velocity and image compression [11],SAR raw data processing[12],etc. In these researches, it has been shown that a successful recovery of a compressively sensed signal depends on the presence of dictionary. These works have made a great contribution to the future research of radar signal processing based on CS. However, all of these applications about imaging algorithm process the echo data under the Nyquist sampling theorem, then reconstructing the targets reflectivity in one dimensional [4,5], or transforming two dimensional signal into a vector and then processing it via CS technique[7,9,10], etc. These methods can't provide practical approaches to simplify radar system and reduce the sampling rate of A/D converter, and at the same time they increase the complexly of radar system. To the author's knowledge, their studies may be more practicality useful if they sample the raw data before radar imaging with lower A/D sampling rate than Nyquist sample's, and then reconstructing the sparse targets at range and azimuth dimensional respectively. This way may be really lighten up sampling burden and improve the resolution of radar system.

In some special applications (the detection of warship, aircraft and spacecraft monitoring, space debris imaging, etc), the main scattering targets distribute in a sparse way over illuminated scene. The number of dominant scatters is much smaller than the number of overall samples. In such a case, SAR echo can be regarded as sparse signal. Thereby, CS sparse reconstructed theory can be used to SAR imaging in these applications. With the consideration of these problems that might happen and the special applications in high resolution SAR imaging, a novel SAR imaging algorithm based on compressive sensing was proposed in this paper. It is outstanding for sampling the raw data with lower A/D sampling rate than the required Nyquist sampling rate, then operating at range and azimuth dimension using CS technique respectively. Simulation denotes this algorithm has a better performance, even with smaller measured samples, than the traditional imaging algorithm based on matched filter method.

Several sections are included in the follows. In Section 2, we will give an introduction to compressed sensing. The deduced of SAR image algorithm based on CS will be discussed in Section 3 in detail. Section 4 presents the results of the algorithm proposed in this paper and conclusions will be given in Section 5.

2. Introduction to compressed sensing

A discrete-time signal x[n] $n=1,2,\cdots,N$ can be represented in some basis

$$\mathbf{x} = \sum_{i=1}^{N} \psi_i \boldsymbol{\theta}_i = \boldsymbol{\Psi\theta} \quad (2.1)$$

Where $\boldsymbol{\Psi} = [\psi_1, \psi_2, \cdots \psi_N]$ is an $N \times N$ basis matrix, $\boldsymbol{\theta}$ is a $N \times 1$ vector with K nonzero elements of weighting coefficients $\boldsymbol{\theta}_i = \langle x, \psi_i \rangle$.And signal $\mathbf{x}$ is called sparse or compressible in $\boldsymbol{\Psi}$ domain with K sparsity.

In compressive sensing theory, the measured signal $\mathbf{y}$ is acquired by projecting $\mathbf{x}$ to the matrix $\boldsymbol{\Phi}$, that is

$$\mathbf{y} = \boldsymbol{\Phi} x = \boldsymbol{\Phi\Psi\theta} \quad (2.2)$$

Where $\boldsymbol{\Phi}$ is a $M \times N$ matrix, $\mathbf{y}$ is a $M \times 1$ vector. Since $M < N$, the recovery of signal $\mathbf{x}$ from the measurements $\mathbf{y}$ is ill-posed in general. But it can get the sparsest solution through the minimal l_0 norm

$$\min \|\boldsymbol{\theta}\|_0 \quad s.t. \quad \mathbf{y} = \boldsymbol{\Phi\Psi\theta} \quad (2.3)$$

Though the l_0 norm can get the signal sparsest, searching the minimum l_0 norm is an N-P problem and it is too sensitive to noise [1]. Consequently, the researchers found out when the matrix $\boldsymbol{\Phi\Psi}$ has the RIP (Restricted Isometry Property)[13], we could solve l_1 norm instead l_0 norm problem which is

$$\min \|\boldsymbol{\theta}\|_1 \quad s.t. \quad \mathbf{y} = \boldsymbol{\Phi\Psi\theta} \quad (2.4)$$

This optimization problem, also known as Basis Pursuit [14], and can be solved with traditional linear programming techniques. At the expense of slightly more measurements, iterative greedy algorithm such as orthogonal matching pursuit (OMP)[15] can recover the signal $\mathbf{x}$ from measurements $\mathbf{y}$.

3. SAR Image algorithm based on CS

3.1 Signal processing model based on random filter CS

The sampling of a signal can be regarded as a matrix operating on the signal. Though the random measure matrix may fulfil RIP with high probability, the system is too difficult to achieve through the mechanism of random measure. Owing to this reason, Joel et al. proposed a new theory named on random filter compressive sensing [15]. The approach captures a signal $\mathbf{s}$ by convolving it with a random-tap FIR filter $\mathbf{h}$ to obtain a compressed representation $\mathbf{y}$ as shown in Fig.1.

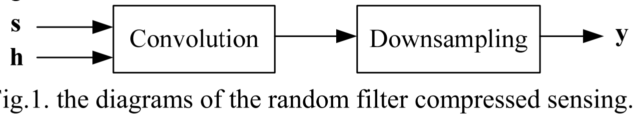


Fig.1. the diagrams of the random filter compressed sensing.

This approach using in signal processing field is as follows. Provided that the transmitted signal $s(t)$ interact with the target described by $u(t)$, the received signal $s_r(t)$ can be written as

$$s_r(t) = \int s(t-\tau) u(\tau) d\tau \quad (3.1)$$

Suppose we get N Nyquist samples of the echo signal with the Δt seconds. But we sample every $D\Delta t$ seconds, where $D = [N/M]$, $M < N$, to obtain the M samples

$$\begin{aligned} y(m) &= s_r(t)\big|_{t=mD\Delta t} \\ &= \int_0^{N\Delta t} s(mD\Delta t - \tau) u(\tau) d\tau \\ &= \sum_{n=1}^{N} s(mD-n) u(n) \end{aligned} \quad (3.2)$$

Where $m = 1, 2, \cdots, M$. Afterwards, the radar signal sequence s implements as a random filter in the sense of [15].That is to say, the transmitted radar waveforms $s(t)$ form a dictionary(the extension of a basis or frame)that is incoherent with time [3].And the lower rate samples y contain sufficient information for reconstructing the signal u corresponding to the Nyquist-rate samples of the reflectivity $u(t)$ via linear programming or a greedy algorithm. The radar system which processed in this way not only eliminate the need of the matched filter, but also slow down the rate of A/D conversion and reduce the amount of the data.

In the following, we apply this approach in reference [15] to SAR signal processing.

3.2. The processing of SAR algorithm based on compressive sensing

Suppose the transmit signal is a linear frequency modulated (LFM), then the echo back from target can be described as

$$\begin{aligned} S_r(\hat{t}, t_m) &= rect\left[(\hat{t} - 2R(t_m)/c) \right] \\ &\times \exp[j\pi\gamma(\hat{t} - 2R(t_m)/c)^2] \\ &\times \exp[-j4\pi R(t_m)/\lambda] \end{aligned} \quad (3.3)$$

Where $\hat{t}$ is the fast time, t_m is the slow time, $rect(\bullet)$ represent the antenna beam pattern in range and azimuth direction, γ is the FM rate $R(t_m) = \sqrt{R_b^2 + (vt_m - x_b)^2}$ is the range from the target position at (R_b, x_b), v is the velocity.

According to the random filter theory (recall section 3.1), we should get fewer sampling data than that of Nyquist sampling firstly. Thereby, we sample the echo signal at $D\Delta\hat{t}$ and $D\Delta t_m$ seconds in range and azimuth dimension. And $P \times Q$ samples can be get, $P = [N_a/D]$, $\mathrm{Q} = [N_\mathrm{r}/D]$, $P < N_a$, $\mathrm{Q} < N_\mathrm{r}$, N_a and N_r represent Nyquist-rate samples in azimuth and range direction respectively. Consequently, the total sampling number of the echo signal reducing from $N_a \times N_\mathrm{r}$ to $P \times Q$.Then we can get aiming at using random filter CS theory

$$\begin{aligned} y(p,q) &= S_r(\hat{t}, t_m; R_0)\Big|_{\hat{t}=qD\Delta\hat{t}, t_m = pD\Delta t_m} \\ &= G \sum_{n_r=1}^{N_r} s_\mathrm{t}(qD - n_r) \delta_r(n_r) \end{aligned} \quad (3.4)$$

Where $q = 1, 2, \cdots, Q$, $p = 1, 2, \cdots, P$, $n_r = 1, 2 \cdots N_r$ $G = \mathrm{re}ct(pD\Delta t_m) \exp\left[-j4\pi R(pD\Delta t_m)/\lambda\right]$.Consequently, only $P \times Q \ll N_\mathrm{a} \times N_r$ are required in random filter CS method. According to random filter CS theory, the range sparse dictionary can be written as

$$\boldsymbol{\Theta}_r = [\Theta_{q1}, \Theta_{q2}, \cdots \Theta_{qn_r} \cdots \Theta_{qN_r}]_{Q\times N_r} \qquad (3.5)$$

Where $\Theta_{qn_r} = rect\left[(qD - n_r)\Delta \hat{t} / t_p\right] \times \exp\{j\pi\gamma[(qD-n_r)\Delta \hat{t}]^2\}$

A linear measurement model should be created in the form of (2.2 aiming at using CS theory. So (3.4) can be written as

$$\mathbf{y}(p) = G(p)\boldsymbol{\Theta}_r\boldsymbol{\theta}_r \qquad (3.6)$$

Where $\boldsymbol{\theta}_r = [\delta_r(1)\cdots\delta_r(N_r)]^T$. According to the CS theory, the sparse solution of (3.6) can be got through solving l_1 norm minimization problem. The reconstructed signal can be written as

$$\begin{aligned} z(p,n_r) &= \widehat{\theta}(n_r) \times \mathrm{rect}(pD\Delta t_m) \times \exp\{-j4\pi R(pD\Delta t_m)/\lambda\} \\ &= \widehat{\theta}(n_r)\sum_{n_a=1}^{Na} G(pD-n_a)\delta_a(n_a) \end{aligned} \qquad (3.7)$$

Where $\mathrm{n_r} = 1,2,\cdots,N_r, n_a = 1,2,\cdots,N_a$, $\widehat{\theta}(n_r)$ is the reconstructed results. Consequently, the azimuth sparse dictionary is

$$\boldsymbol{\Xi}_\mathbf{a} = \left[\Xi_1, \Xi_2 \cdots \Xi_{n_a} \cdots \Xi_{N_a}\right]_{P\times N_a} \qquad (3.8)$$

Where $\Xi_{pn_a} = G\left[(pD-n_a)\Delta t_m\right]$

Using the CS technique, sparse reconstruction linear measurement model along the azimuth dimension can be written as

$$\mathbf{z}(n_r) = \widehat{\theta}(n_r)\Xi_a\boldsymbol{\theta}_a \qquad (3.9)$$

Where $\boldsymbol{\theta}_a = [\delta_a(1), \delta_a(2), \cdots \delta_a(N_a)]^T$ represents the scattering reflectivity along the azimuth direction.

After reconstructing the spares results of azimuth dimension using the minimum l_1 norm, we can get the focus results of the SAR imaging. The block diagram of SAR imaging algorithm based on compressed sensing is given in Fig.2

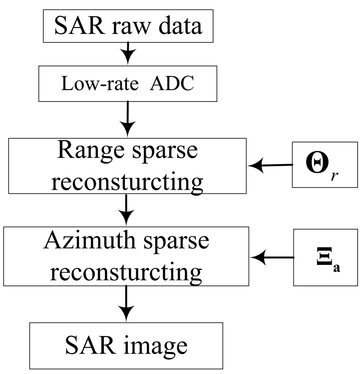


Fig.2.The block diagram of SAR imaging algorithm

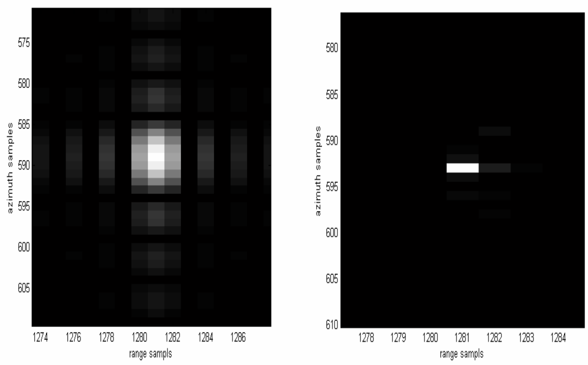
4. Simulation and discussions

To demonstrate the validity of the algorithm based on compressed sensing derived in the paper, the simulation of the nine point targets was carried out. The parameters used are shown in Table 1

Table 1 System Parameters

System Parameters	Numerical Value
bandwidth	100 MHz
PRF	500 Hz
Velocity	100 m/s
beam-width	15^0
Elevation angle	45^0
Pulse width	10 us
Platform height	1000m
Scene size(range×azimuth)	600m×200m

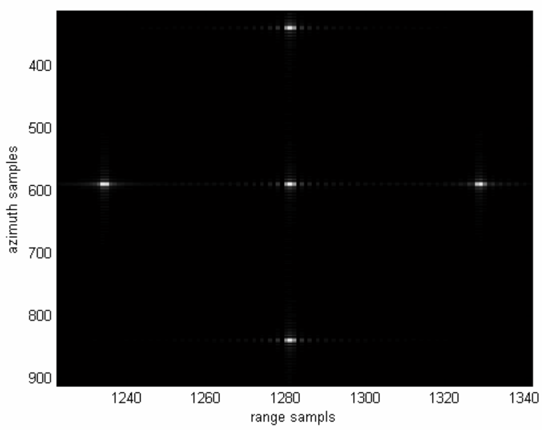
The sample rate D (as shown in (3.4)) of range and azimuth is 5. Using recover imaging processing algorithm as shown in Fig.2, the processing result of one point target which position is （1000，0）is shown in Fig.3(b).And Fig.3(a) is the traditional SAR imaging result. Compared Fig.3(b) to Fig.3(a), we can obviously observe that the target position are clearly reconstructed. In addition, the sidelobe of simulated results using the proposed method is far less than traditional SAR imaging results, and the CS-based imaging quality performs better effects than traditional processing method.



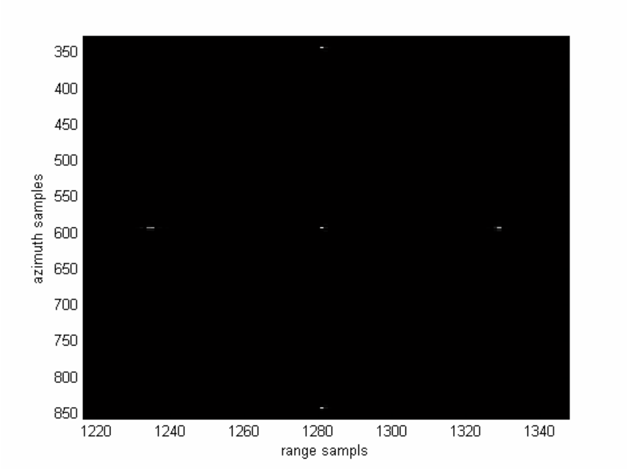
(a) traditional method (b) CS-based method

Fig.3.compressed sensing imaging results (enlarged)

We also reconstruct five scatters perfectly as shown in Fig.4 (b) using the proposed algorithm. And the targets position are located :(1000，0),(950，0),(1050，0),(1000，50),(1000,-50).The sample rate D (as shown in (3.4)) of range and azimuth is 5.The traditional SAR imaging results are shown in Fig.4(a).



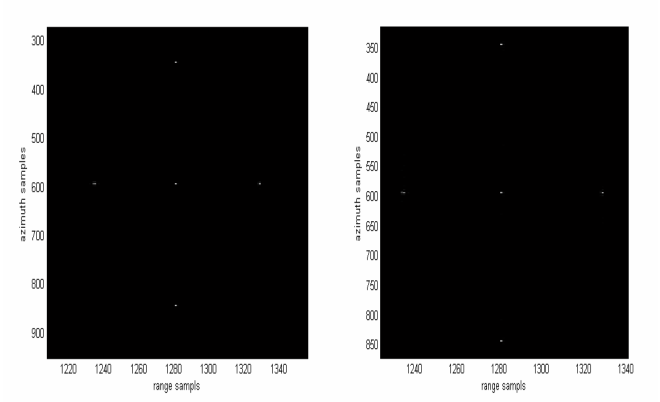
(a)traditional method



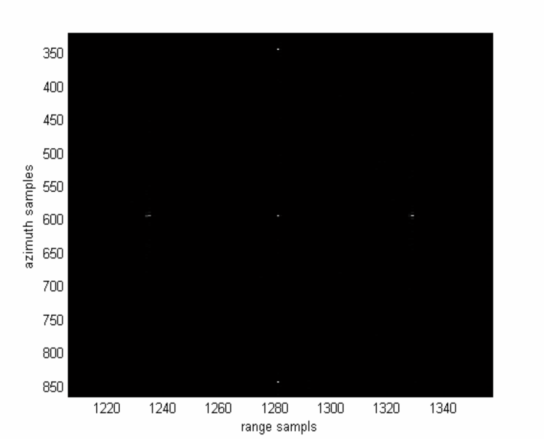
(b)CS-based method(noise free)

Fig.4.Simulation results of five point targets(enlarged)

Fig.5 show the different simulation results at different SNR. From the results we can see that the performance is worse when the SNR is down. While SNR is -5dB, we could still get the results. We may get the conclusion that the algorithm can reconstruct at low SNR.



（a） SNR=10dB （b） SNR=0dB



（c） SNR=-5dB

Fig.5. reconstruction results at different SNR(enlarged)

5 Conclusions

This paper presents a novel 2-D SAR imaging algorithm based on CS theory. It is outstanding for sampling the raw data with lower sampling rate the Nyquist sampling rate, and then operates at range and azimuth dimensional respectively. Compared with traditional SAR imaging algorithm, the proposed imaging algorithm significantly suppresses the sidelobe of imaging results, and greatly improves the imaging performance of SAR when the target space is sparse. Simulated results show that the proposed algorithm has a better performance than the traditional imaging algorithm, even with smaller measured samples, and also indicate that the algorithm is robustness with existence of serious noise.

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