A Novel Two Dimensional Imaging Algorithm Based on Compressed Sensing for Multi-Channel SAR

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Abstract

In this paper, a novel synthetic aperture radar (SAR) two-dimensional (2-D) imaging algorithm which named CSMC is proposed based on compressed sensing (CS) and multi-channel (MC) SAR system. In particular, the algorithm operates in range and azimuth dimensions via CS technique, respectively. This new algorithm simultaneously provides a high resolution and wide-swath 2-D map of the spatial distribution of targets with a significant reduction in the number of data samples beyond the Nyquist theorem and with an implication in simplification of radar architecture. The simulation results show that this new imaging scheme presents many important applications and advantages which include higher resolution, less sampled data and higher noise immunity.

1 Introduction

Synthetic aperture radar (SAR) is a radar imaging technology that is capable of producing high resolution images of the stationary surface targets. The main advantages of SAR are that it can reduce the effects of clouds and fog and allow them to be independent of external sources for imaging, having day and night and all-weather imaging capability. Traditional compression of SAR data claims that one must sample at least two times faster than the signal bandwidth while capturing it without losing information. Thereby there are large amounts of onboard data that have to be stored and it inevitably results in complex computation and expensive hardware.

On the other hand, future SAR will be required to produce high-resolution image over a wide area of surveillance. However, minimum antenna area constraint makes it a contradiction to obtain both high azimuth resolution and wide-swath simultaneously, which is derived form the inconsistent requirements for pulse repetition frequency (PRF).In the last past years, multi-channel SAR was proposed to address the question on how to achieve high resolution and wide-swath SAR images at the same time[1]. Multi-channel SAR systems gather additional information with their multiple receive apertures what enables to image wide swaths with a high geometric resolution. The recently introduced theory of compressed sensing (CS) is a new concept allowing recovery of signals that have been sampled below the traditional Nyquist sampling rate. In this new framework, it uses a low-dimensional, nonadaptive, linear projection to acquire an efficient representation of a sparse signal with just a few measurements, so as to greatly reduce the sampling rate and enhance the data rate. Because of its compressed sampling ability, compressed sensing has found many applications in radar and remote sensing, and other fields. R.Baraniuk *et al*. proposed the radar imaging system based on compressed sensing for the first time [2]. The paper [3] uses CS in along-track interferometric SAR imaging.

In this paper, we introduce a novel synthetic aperture radar two-dimensional(2-D) imaging algorithm which named C-SMC based on compressed sensing (CS) theory and multi-channel (MC) SAR system. The key idea in our approach is to use CS to reconstruct 2-D target in the range dimension and azimuth dimension, respectively. Meanwhile, it provides the potential to achieve higher resolution between targets and to reduce on-board storage constraints. More importantly, our method does not use a matched filter and enhances some of these suggestions and provides a proper framework along with general reconstruction techniques.

2 Compressed Sensing

According to the compressed sensing theory[4], there are three important ingredients:sparse signal representation, measurement operator, and sparse reconstruction algorithms. Consider a discrete signal expressed as a vector $\boldsymbol{x} \in \mathbb{C}^N$ of length N. Suppose $\boldsymbol{x}$ is *K-sparse* if at most $K \ll N$ of its coefficients are nonzero in a basis or more generally a frame $\boldsymbol{\Psi}$, so that $\boldsymbol{x} = \boldsymbol{\Psi s}$, where $\boldsymbol{\Psi} \in \mathbb{C}^{N\times N}$ is a matrix and $\boldsymbol{s} \in \mathbb{C}^N$ is a vector. The signal is acquired through linear projections:

$$\boldsymbol{y} = \boldsymbol{\Phi x} = \boldsymbol{\Phi\Psi s} = \boldsymbol{\Theta s} \tag{1}$$

where $\boldsymbol{y} \in \mathbb{C}^M$ is the measurements vector and $\boldsymbol{\Phi} \in \mathbb{C}^{M\times N}$ is the measurement matrix with $M < N$. Since

$M < N$, the recovery of signal $\boldsymbol{x}$ from the measurements vector is ill-posed in general. But when the matrix $\boldsymbol{\Theta}$ has the Restricted Isometry Property (RIP)[5], it is possible to reconstruct $\boldsymbol{x}$ from a set of $M = O(K \log(N/K))$ linear measurements. So the signal $\boldsymbol{x}$ can be perfectly recovered via its coefficients $\boldsymbol{s}$ with high probability, by solving the following l_1 minimization problem [5]:

$$\hat{\boldsymbol{s}} = \arg\min \|\boldsymbol{s}\|_1 \quad subject\ to\ \boldsymbol{y} = \boldsymbol{\Phi\Psi s} = \boldsymbol{\Theta s} \tag{2}$$

The optimization problem (2) is often known as Basis Pursuit (BP) and Orthogonal Matching Pursuit (OMP), etc.

3 CS Applied for Multi-Channel SAR

3.1 Signal model of multi-channel SAR

Azimuth multi-channel SAR imaging system includes single transmitter multiple azimuth beams (STMAB) and multiple transmitters multiple azimuth beams (MTMAB) system. Figure 1(an example of three antennas) shows a simple diagram of stripmap MTMAB SAR and how received data are placed in a three-dimensional (3-D) signal model. The range is the direction of signal propagation and the azimuth is the direction parallel to the flight path.

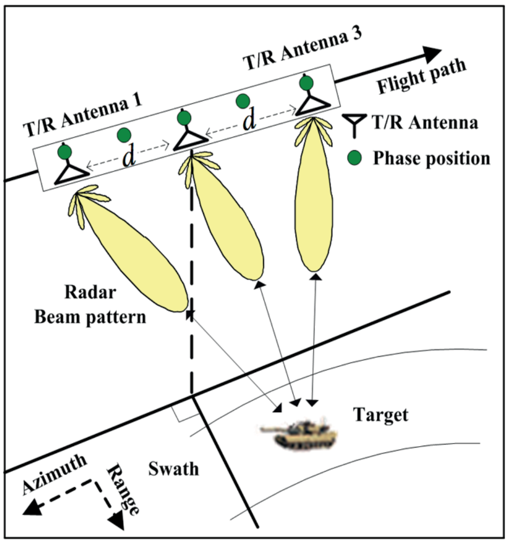


Figure 1: Multi-Channel SAR data collection in 3-D

Compared to a single-aperture system, each antenna of multi-channel SAR system receives and transmits signals. The outputs from each channel will afterwards be got together during processing. Multiple receivers gather for the same pulse repetition frequency (PRF) in azimuth dimension, thereby ensuring constant performance over a clearly extended PRF range. Suppose K is antenna number, according to the theory of multi-channel SAR, there are $2K-1$ equivalent phase centre positions. The antenna positions are governed by the spacing d of the K receivers in combination with the distance between subsequent pulses given by the sensor velocity v and PRF. Consequently, a uniform sample distribution is obtained only if the following timing requirement is fulfilled [6]:

$$d = \frac{2v}{(2K-1)PRF} \tag{3}$$

Thus, the virtual uniform linear array of multi-channel SAR is composed of the equivalent phase centre positions. Suppose the *k-th* antenna transmits signals and the *l-th* antenna receives echo signals. As shown in Figure 1, the sum of the range from the transmitter and receiver to the target can be written as

$$\begin{aligned} R_{kl}(\eta) &= R_k(R_b, x, \eta) + R_l(R_b, x, \eta) \\ &= \sqrt{R_b^2 + (v\eta - kd - x)^2} \\ &+ \sqrt{R_b^2 + (v\eta - ld - x)^2} \end{aligned} \tag{4}$$

where (R_b, x) is the coordinate of the target, η is the slow time. Suppose the transmitted signal is linear frequency modulated (LFM) signal which can be described as

$$s_T(\tau) = \mathrm{rect}\left(\frac{\tau}{T_p}\right) \exp\left\{j2\pi f_c\tau + j\pi k_r\tau^2\right\} \tag{5}$$

where T_p is the pulse duration, τ is the fast time, f_c is the carrier frequency, k_r is the chirp rate and $\mathrm{rect}(\cdot)$ is the stand for the unit rectangular function. After mixing down and quadrature demodulation, the received radar signal is given by

$$\begin{aligned} s_R(\tau, \eta) &= \mathrm{rect}\left(\frac{\tau - R_{kl}(\eta)/c}{T_p}\right) \\ &\times \exp\left\{j\pi k_r\left(\tau - \frac{R_{kl}(\eta)}{c}\right)^2 - j\frac{2\pi R_{kl}(\eta)}{\lambda}\right\} \end{aligned} \tag{6}$$

where c is the speed of light, and λ is the wavelength of the transmitted signal.

3.2 CSMC SAR Imaging Algorithm

Baraniuk and Steeghs[2] proposed a new application that named random filtering for SAR based on compressed sensing. This approach does not use matched filters. With the actual radar system, random filtering can be performed by the following method. Suppose $s_T(\tau)$ is the transmitted signal and the target is described by $u(\tau)$, then the received signal $s_R(\tau)$ can be written as

$$s_R(\tau) = \int s_T(\tau - \xi)u(\xi)d\xi \tag{7}$$

Consider a target reflectivity generated from N Nyquist-rate samples $x(n)$ via $x(n) = u(\Delta t)$,$n = 1, \cdots, N$.We sample the received radar signal $s_R(\tau)$ not every Δt seconds but rather every $D\Delta t$ seconds, where $D = [N/M]$ and $M < N$, to obtain the M samples,$m = 1, \ldots, M$

$$\begin{aligned} y(m) &= s_R(\tau)|_{\tau=mD\cdot\Delta t} \\ &= G\int_0^{N\Delta t} s_T(mD\cdot\Delta t-\xi)u(\xi)\,d\xi \\ &= G\sum_{n=1}^{N} s_T(mD\cdot\Delta t-n)\int_{(n-1)\Delta t}^{n\Delta t} u(\xi)d\xi \\ &= G\sum_{n=1}^{N} s_T(mD-n)x(n) \end{aligned} \tag{8}$$

where G represents attenuation due to propagation and reflection, $s_T(n)$ is the discrete transmitted signal. The low-rate samples y contain sufficient information to reconstruct the signal x corresponding to the Nyquist-rate samples of the reflectivity $u(\tau)$ via linear programming or a greedy algorithm[2].The measurement vector is reduced from N to M via random filtering.

Using the concepts of random filtering, we can apply this approach to multi-channel SAR system. Firstly, the measurement matrix of range dimension is constructed in terms of equation (8). Suppose D_r represents the down-sampling times in the range direction, the range measurement matrix can be expressed as

$$\begin{aligned} \mathbf{\Phi_r}(m,n) &= s_T(mD_r-n) \\ &= \mathrm{rect}\left(\frac{mD_r-n}{T_p}\right)\exp\left\{j\pi k_r(mD_r-n)^2\right\} \end{aligned} \tag{9}$$

where $\mathbf{\Phi_r} \in \mathbb{C}^{M\times N}$,$M = N/D_r$,$m = 1,\ldots M$,$n = 1,\ldots N$. After the targets being reconstructed in range dimension via CS, the signal can be approximated as

$$s_{cs}(\tau,\eta) \approx \sin c\left(\tau-\frac{R_{kl}(\eta)}{c}\right)\exp\left\{-j\frac{2\pi R_{kl}(\eta)}{\lambda}\right\} \tag{10}$$

where $\sin c(\cdot)$ is the Sinc function. The second factor of (10) is the Doppler phase factor. Suppose D_a represents the down-sampling times in azimuth direction, the azimuth measurement matrix can be given by

$$\begin{aligned} \mathbf{\Phi_a}(q,p) &= \exp\left\{-j2\pi\frac{R_{kl}(q,p)}{\lambda}\right\} \\ &= \exp\left\{-j2\pi\frac{\sqrt{R_b^2+[v(qD_a-p)-kd-x]^2}}{\lambda}\right. \\ &\quad \left. -j2\pi\frac{\sqrt{R_b^2+[v(qD_a-p)-ld-x]^2}}{\lambda}\right\} \end{aligned} \tag{11}$$

where $\mathbf{\Phi_a} \in \mathbb{C}^{Q\times P}$,$p = 1,\ldots P$ is the Nyquist sampling sequence in azimuth, $Q = P/D_a$, $q = 1,\ldots Q$ is the down-sampling sequence.After constructing $\mathbf{\Phi_r}$ and $\mathbf{\Phi_a}$, the equation (2) can solve by OMP or BP in range and azimuth dimension, respectively.Finally, we can get the C-SMC SAR 2-D images.

4 Experimental Results

Simulated data have been used to validate the algorithm in this paper. We use the solution method of optimization directly as OMP. In following experiments, we set the simulation radar parameters as listed in Table 1.

Table 1: Simulated Radar Parameters

Parameters	Value
Transmitted antennas Numbers	3
Received antennas Numbers	3
Carrier frequency	$10GHz$
Transmitted signal bandwidth	$150MHz$
Platform height	$20km$
Platform velocity	$1350m/s$
Pulse duration	$5\mu s$
PRF	$270Hz$
Range undersampling	5 times
Azimuth undersampling	4 times

4.1 Point scatterers simulation

The 9 point targets are reconstructed via traditional SAR imaging algorithm and CSMC algorithm, respectively. The results are shown in Figure 2 and Figure 3. Figure 2 shows the SAR imaging results with traditional Nyquist theory. The results of the CSMC algorithm are shown in Figure 3. Simulation demonstrates that the proposed algorithm can give an exact recovery of the reflectivity function although the amount of data is reduced by 5×4 times.

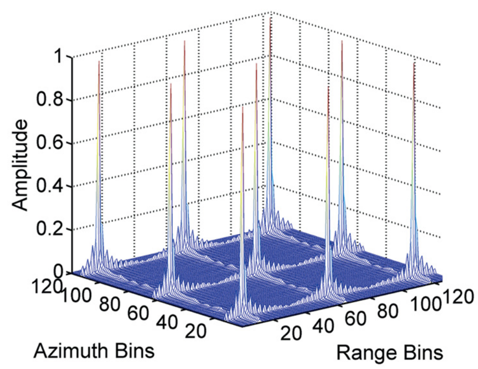


Figure 2: 3-D reflectivity with traditional reconstruction.

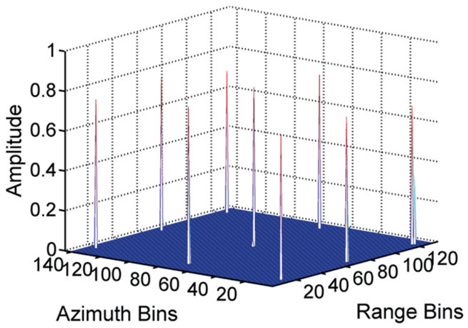
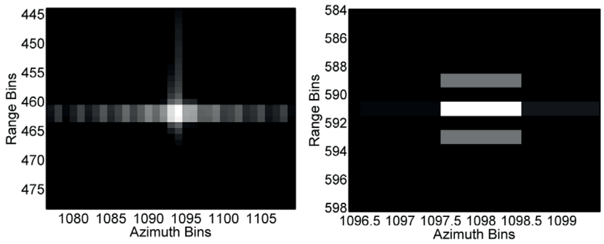


Figure 3: 3-D reflectivity with the proposed CSMC.

4.2 Resolution analysis

We show an important capability of this method that is high resolution. This experiment sets the traditional radar azimuth resolution of $1.5m$. Let the coordinates of the targets are $(0m, 0m)$, $(0m, 1m)$ and $(0m, -1m)$. In this case, the traditional SAR imaging algorithm can not distinguish the 3 point targets which their azimuth distance is 1 meter as the Figure 4(a) shows. On the contrary, the proposed C-SMC can distinguish the targets as the Figure 4(b) shows. This experiment illustrates CS theory has an enormous potential application in improving radar resolution.



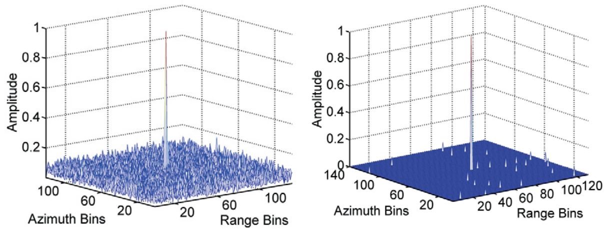
(a) (b)

Figure 4: Comparison of resolution obtained via two different methods. (a) Reconstructed by traditional imaging method. (b) Reconstructed by our proposed method.

4.3 Noise immunity

The results in low signal to noise ratio (SNR) with traditional reconstruction and CSMC reconstruction are shown in the following figures. Figure 5(a) and Figure 5(b) illustrate their respective results in the presence of -20dB of additive white Gaussian noise (AWGN) using traditional imaging algorithm and the CSMC algorithm.

Compare the two figures, we can find out targets can be well reconstructed in presence of serious noise using C-SMC algorithm and the performance is better than the traditional method. Therefore, the proposed algorithm is strongly immune to noise interference and has the characteristic of high noise immunity.



(a) (b)

Figure 5: Comparison of Noise immunity via two different methods. (a) Traditional algorithm with -20dB AWGN. (b) CSMC reconstruction with -20dB AWGN.

5 Conclusion

In this paper, a novel 2-D SAR imaging algorithm is proposed based on constructing measurement matrixes in range and azimuth dimension using compressed sensing techniques, respectively. And the multi-channel technology is used in this algorithm to resolve the contradiction between high resolution and wide-swath. The CSMC imaging method not only reduces the amount of echo data but also does not need a matched filter at the receiver. The simulation experiments verify the validity of the proposed algorithm which has higher resolution, less sampled data and higher noise immunity.

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